

1. A particle P of mass 0.5 kg falls vertically from rest. After t seconds it has speed $v \text{ m s}^{-1}$. A resisting force of magnitude $1.5v$ newtons acts on P as it falls.

(a) Show that $3v = 9.8(1 - e^{-3t})$. (8)

(b) Find the distance that P falls in the first two seconds of its motion. (5)

2.

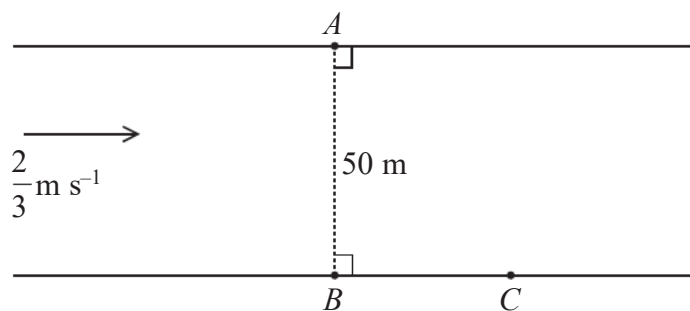


Figure 1

A river is 50 m wide and flows between two straight parallel banks. The river flows with a uniform speed of $\frac{2}{3}$ m s⁻¹ parallel to the banks. The points *A* and *B* are on opposite banks of the river and *AB* is perpendicular to both banks of the river, as shown in Figure 1.

Keith and Ian decide to swim across the river. The speed relative to the water of both swimmers is $\frac{10}{9}$ m s⁻¹.

Keith sets out from *A* and crosses the river in the least possible time, reaching the opposite bank at the point *C*. Find

(a) the time taken by Keith to reach *C*, (2)

(b) the distance *BC*. (2)

Ian sets out from *A* and swims in a straight line so as to land on the opposite bank at *B*.

(c) Find the time taken by Ian to reach *B*. (4)

3.

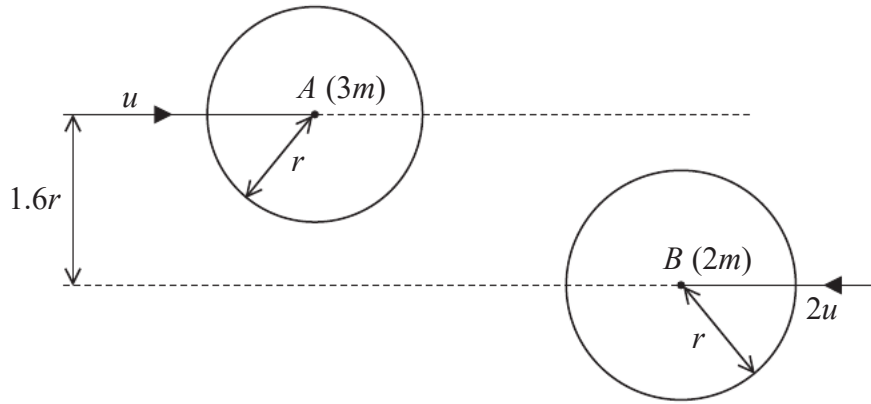


Figure 2

Two smooth uniform spheres A and B , of equal radius r , have masses $3m$ and $2m$ respectively. The spheres are moving on a smooth horizontal plane when they collide. Immediately before the collision they are moving with speeds u and $2u$ respectively. The centres of the spheres are moving towards each other along parallel paths at a distance $1.6r$ apart, as shown in Figure 2.

The coefficient of restitution between the two spheres is $\frac{1}{6}$.

Find, in terms of m and u , the magnitude of the impulse received by B in the collision.

(10)



4.

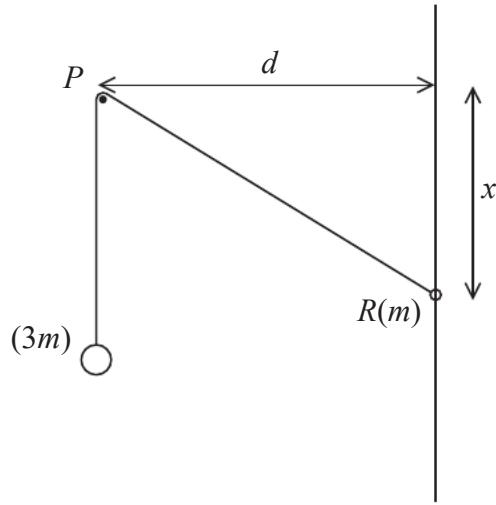


Figure 3

A small smooth peg P is fixed at a distance d from a fixed smooth vertical wire. A particle of mass $3m$ is attached to one end of a light inextensible string which passes over P . The particle hangs vertically below P . The other end of the string is attached to a small ring R of mass m , which is threaded on the wire, as shown in Figure 3.

- (a) Show that when R is at a distance x below the level of P the potential energy of the system is

$$3mg \sqrt{(x^2 + d^2)} - mgx + \text{constant} \quad (4)$$

- (b) Hence find x , in terms of d , when the system is in equilibrium. (3)

- (c) Determine the stability of the position of equilibrium. (3)

6. A particle P of mass m kg is attached to the end A of a light elastic string AB , of natural length a metres and modulus of elasticity $9ma$ newtons. Initially the particle and the string lie at rest on a smooth horizontal plane with $AB = a$ metres. At time $t = 0$ the end B of the string is set in motion and moves at a constant speed U m s⁻¹ in the direction AB . The air resistance acting on P has magnitude $6mv$ newtons, where v m s⁻¹ is the speed of P . At time t seconds, the extension of the string is x metres and the displacement of P from its initial position is y metres.

Show that, while the string is taut,

(a) $x + y = Ut$ (2)

(b) $\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 9x = 6U$ (5)

You are given that the general solution of the differential equation in (b) is

$$x = (A + Bt)Ue^{-3t} + \frac{2U}{3}$$

where A and B are arbitrary constants.

(c) Find the value of A and the value of B . (5)

(d) Find the speed of P at time t seconds. (2)



Question 6 continued

Lined writing area for the student's answer to Question 6.

7. [In this question \mathbf{i} and \mathbf{j} are perpendicular unit vectors in a horizontal plane]

A small smooth ball of mass m kg is moving on a smooth horizontal plane and strikes a fixed smooth vertical wall. The plane and the wall intersect in a straight line which is parallel to the vector $2\mathbf{i} + \mathbf{j}$. The velocity of the ball immediately before the impact is $b\mathbf{i} \text{ m s}^{-1}$, where b is positive. The velocity of the ball immediately after the impact is $a(\mathbf{i} + \mathbf{j}) \text{ m s}^{-1}$, where a is positive.

(a) Show that the impulse received by the ball when it strikes the wall is parallel to $(-\mathbf{i} + 2\mathbf{j})$. (1)

Find

(b) the coefficient of restitution between the ball and the wall, (8)

(c) the fraction of the kinetic energy of the ball that is lost due to the impact. (3)
